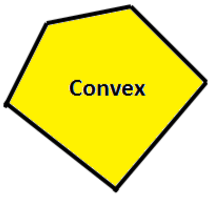
A Polygon Review for Teachers

<u>A Review of Polygons --- properties and areas</u>:

1. **<u>Define polygons</u>:** The word "polygon" derives from the Greek *poly*, meaning "many," and *gonia*, meaning "angle." Review, find examples or draw pictures of polygons, and then discuss with students (and record explanations) the following:

    - **What is a polygon?** (<u>answer</u>: *it is a flat, 2- dimensional closed geometric shape bounded by 3 or more straight line segments (called sides), and an equal number of points (called vertices) --- note: a circle is not a polygon because it does not have straight sides).*

    - A **vertex** (plural "vertices") is a corner. A side joins one vertex with another.



    - An **angle** is the amount of turn between two straight lines that have a common end point (the vertex).

    - A **convex** polygon has no angles pointing inwards. More precisely, no internal angle can be more than 180°.

    - If any internal angle is greater than 180° then the polygon is **concave**. (*Think: concave has a "cave" in it*)

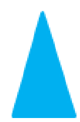
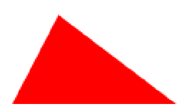
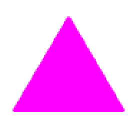
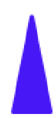
    - For two-dimensional figures, any side can be a **base**. Typically, however, the bottom side, on which the polygon 'sits,' is called the base.

    - **Equilateral** means all sides are equal in length**. Equiangular** means all angles are equal in measurement.

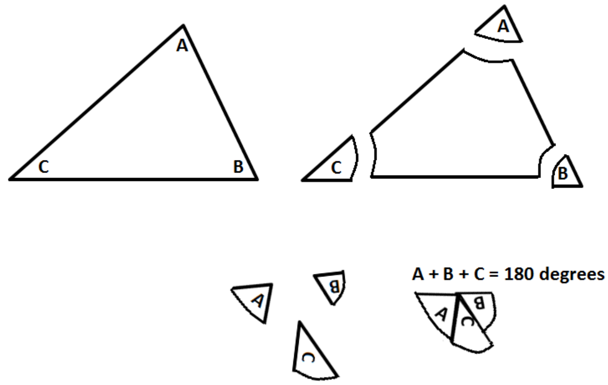
    - A **regular** polygon has all angles equal and all sides equal, otherwise it is **irregular**.

- **Polygons are named for their number of sides and angles:**
    - 3: Triangle
    - 4: Quadrilateral
    - 5: Pentagon
    - 6: Hexagon
    - 7: Heptagon
    - 8: Octagon
    - 9: Nonagon
    - 10: Decagon
    - 11: Undecagon
    - 12: Dodecagon
    - n: n-gon

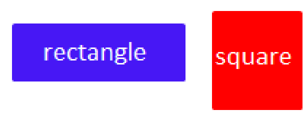
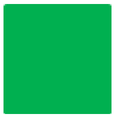
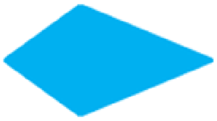
2. **<u>Types of triangles</u>**: (a polygon with 3 sides)…. See examples below (remember some triangles belong to more than one category of triangle)

Right Triangle	Has a 90 degree angle	
Acute Triangle	Each angle < 90 degrees	
Obtuse Triangle	One angle > 90 degrees	
Scalene Triangle	Sides all different lengths	
Equilateral Triangle	Sides all equal in length	
Isosceles Triangle	≥ two sides equal in length	

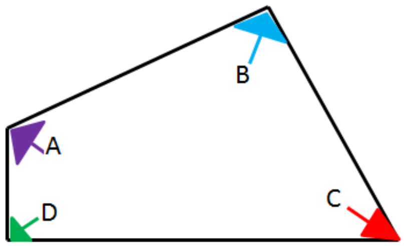
3. **<u>The sum of the interior angles in a triangle always equals 180 degrees.</u>**
   How can you show the sum of the interior angles in any triangle is 180°? See example below:



4. **<u>Types of quadrilaterals</u>** (a polygon with four sides) …. See examples below (remember some quadrilaterals belong to more than one category of quadrilaterals). **<u>There are 5 special quadrilaterals:</u> Rectangles, Squares, Parallelograms, Rhombuses, and Trapezoids.** (Note: a kite is a *Rhombus* if all sides are equal in length and a kite is a *Square* if all sides & angles are equal).

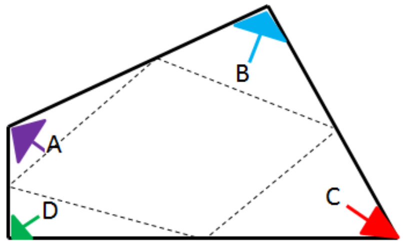
Rectangles, squares	Have pairs of parallel opposite sides that are the same lengths, and all four corners are right angles	
Squares	A square is a special rectangle where all four sides are equal in length	
Parallelograms	Opposite sides are a parallel to each other and equal in length. Rectangles and squares are special examples	
Rhombuses	Are equilateral parallelograms	
Trapezoids	2 opposite parallel sides, while the other 2 sides are not parallel	
Kite	Opposite angles and adjacent sides are equal in length	

5. **Interior angles of quadrilaterals equal 360 degrees:**
How can you show the sum of the interior angles in any quadrilateral is 360°?
See folding example below:

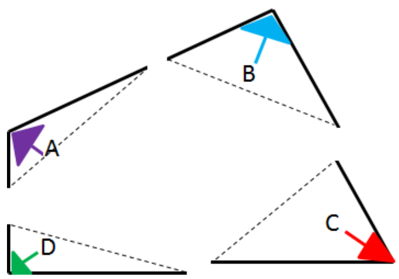


Color or label the vertices of a quadrilateral.

→



Cut off the corners, and then fit the vertices together showing they all fit around a point. The sum of the interior angles in any quadrilateral is 360°.



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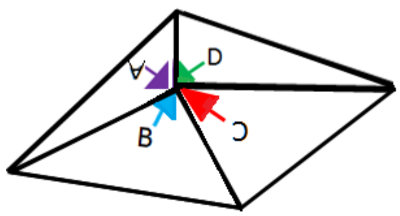
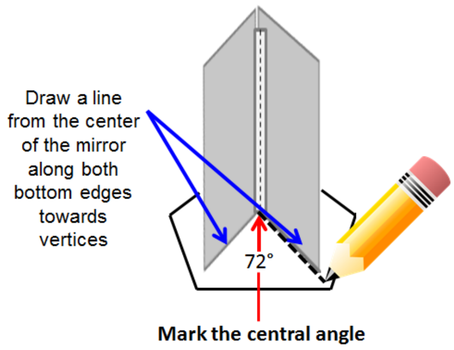


6. **Interior Angles of Regular Pentagons:**

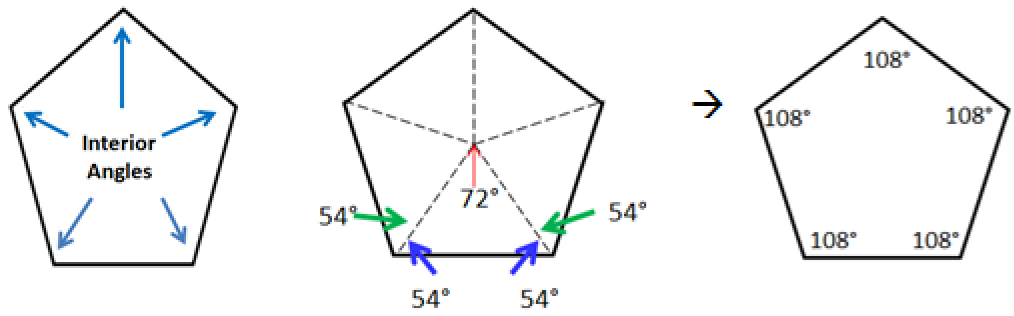
To find the central angle

use a hinged mirror →



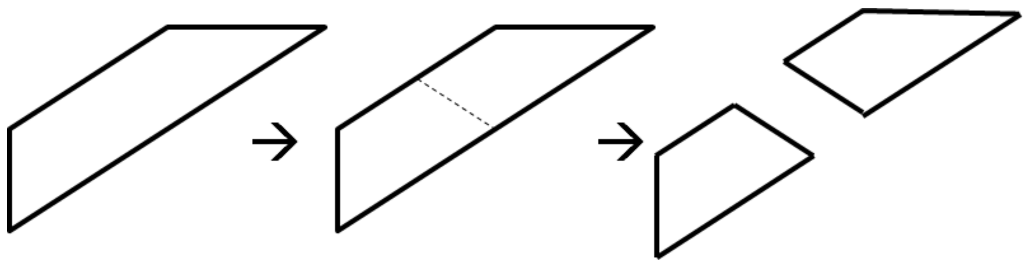
The central angle of a regular pentagon is found also by noticing it can be split into five regular triangles that meet up in the center whose angles add to 360 degrees.

That means one central angle is → 360°/5 = 72°



7. **Polygons from other polygons:**
   - How can you make one type of triangle out of another type? (pass out various paper triangles, fold them, check) --- e.g., fold an isosceles triangle in half to make two smaller right triangles.

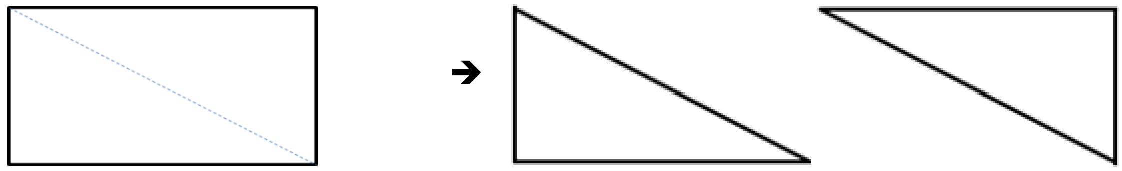
   - How can you make one type of quadrilateral out of another type? (Pass out paper quadrilaterals, fold them, check) --- e.g., see illustrations below:



   - How can you make a triangle from a quadrilateral? Vice versa? (e.g., experiment with folding different polygons to make them into other polygons). Here is an illustration:

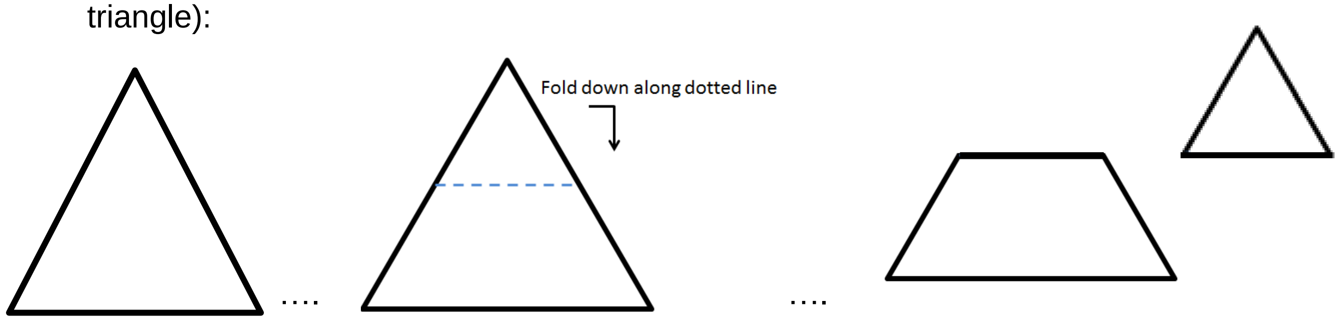
Quadrilateral into a triangle(s) →

Cut a rectangle along diagonal line to form 2 triangles.



Triangles into a quadrilateral →

Fold an equilateral triangle as shown below to create a trapezoid (& another triangle):

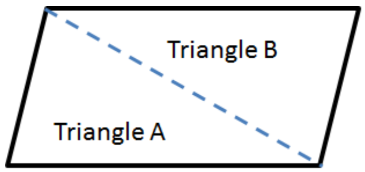


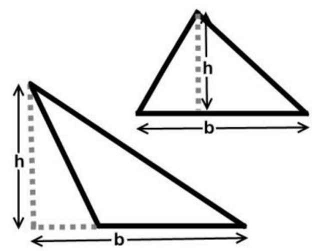
- If you have one shape, such as a pentagon, could you fold it to make other polygons? (Pass out a variety of different paper shapes, fold them, check).
- Can you make one larger polygon from smaller polygon shapes? (Pass out paper shapes, fold them, check).

8. Define Area:

- The **area of a polygon** is the number of square units inside that polygon.

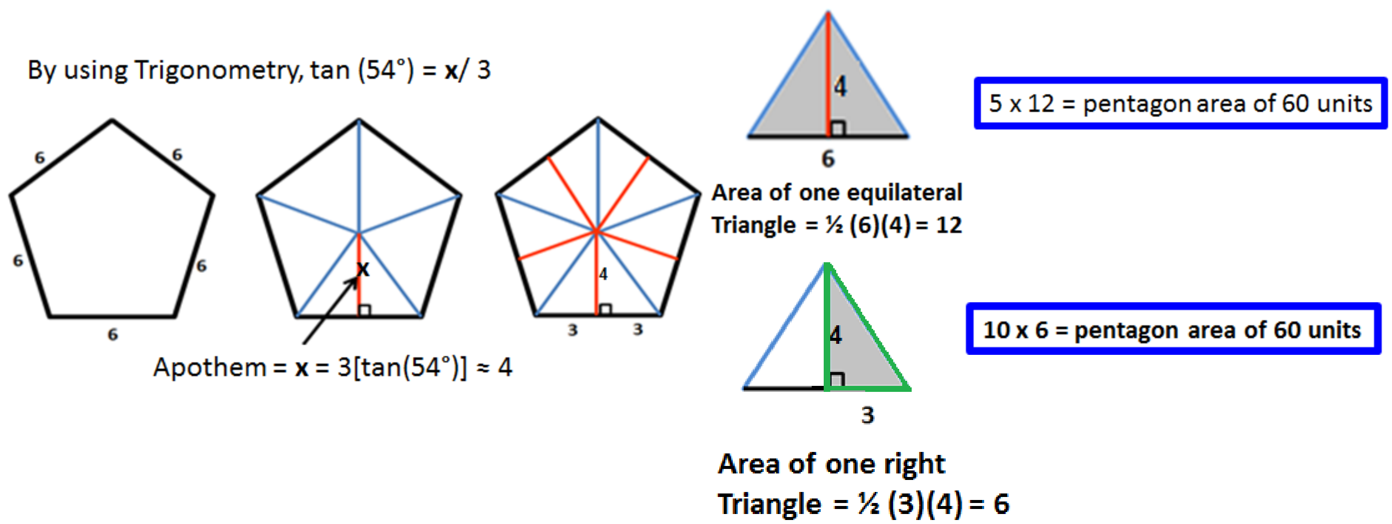
- The **area of a triangle** is ½ (base)(height). This is determined from the fact that a parallelogram can be divided into 2 triangles. The area of a parallelogram is (base)(height). For instance, in the parallelogram below, the area of each triangle equals ½ the area of the parallelogram. The base and height of a triangle must be perpendicular to each other.





- How could you **find the area of a regular pentagon**?

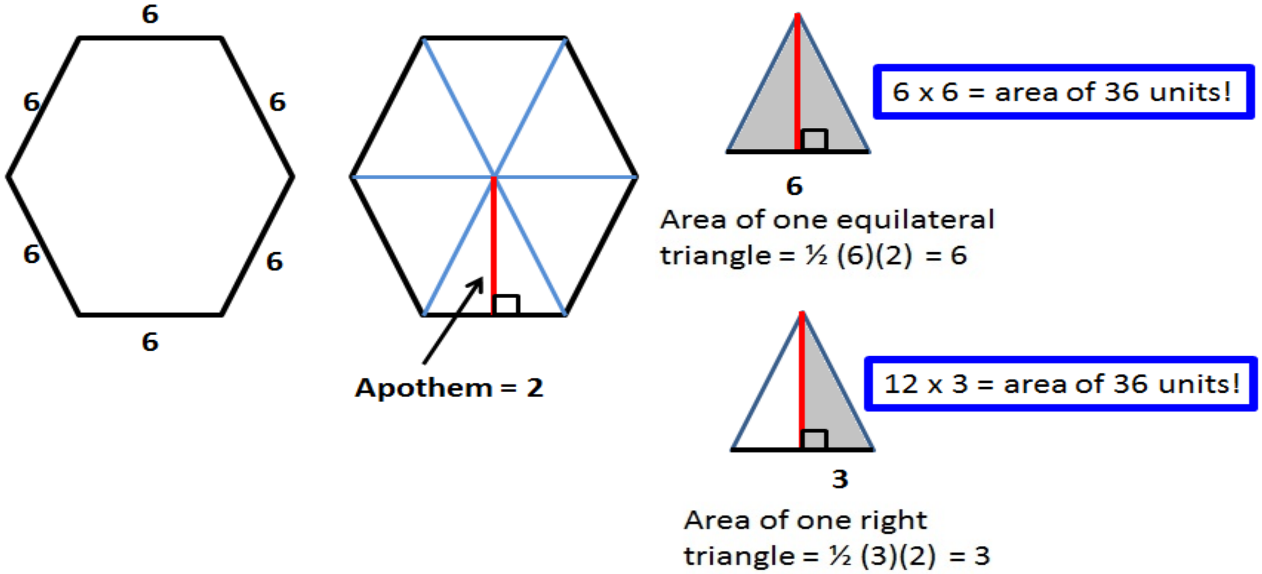
(answer: one way is to deconstruct it into triangles, find the areas of those, and then add their areas to reconstruct the original larger shape). For example let's find the area of a regular pentagon, having each side 6 units in length and an apothem of approximately 4 units (the apothem is the line from the center of the pentagon to a side, intersecting the side at a 90º right angle) —see illustration below where the pentagon is split into 5 identical equilateral triangles, and then into 10 equivalent right triangles:



- How could you **find the area of a regular hexagon**?

(answer: again, one way is to deconstruct it into triangles, find the areas of those, and then add their areas to reconstruct the original larger shape). For example:

Use trigonometry to find the value of the apothem in a regular pentagon and regular hexagon, for instance, when the side value is known (not required to teach at this grade level, but just noted as a way to find the approximate apothem value).



9. What is meant by surface area? (answer: The **surface area** of a solid object is a measure of the total area that the surface of an object occupies). The surface area for polyhedra (i.e., objects with flat polygonal faces), is the sum of the areas of its faces. The surface area of curved surfaces involves more than just adding up areas of flat surfaces.

- Ask students to contemplate: How do soccer balls roll when their surface area is not smooth: it is made up of hexagons and pentagons? How might you estimate the surface area of a soccer ball? What strategies could you use?